INDIAN STATISTICAL INSTITUTE, CHENNAI CENTRE

PROBABILITY (FINAL)

MSTAT FIRST YEAR

Date 26/11/2016

Max: 60
Time: 3Hours

1. i) Let $A_n = [0, 1 - \frac{1}{n}]$ if n is even and $A_n = [-1, \frac{1}{n}]$ if n is odd. Find $limsupA_n$ and $liminfA_n$. Are they equal?
   ii) If $P(A_n) = 1$ for $n = 1, 2, 3, ...\infty$. Compute $P(\cap A_n)$. [2 + 2 + 3]

2. Let $f(x,y) = \begin{cases} = 8xy & if \quad 0 \leq x \leq y \leq 1 \\ = 0 & otherwise \end{cases}$
   i) Are the two random variables independent?
   ii) Let $F(x,y) = 1$ if $x + y \geq 0$ and $F(x,y) = 0$ if $x + y < 0$. Examine whether F represents joint distribution function of (X, Y). [4 + 4]

3. Let X follow double exponential with density function $f(x) = \frac{1}{2}e^{-|x|}$,
   i) Show that the distribution function F of X is symmetric, that is $F(x) + F(-x) = 1$ for all $x \in R$.
   ii) Show that the characteristic function of X is real valued. Also explicitely evaluate it. [2 + 3 + 5]

4. Let F_n,$n = 1, 2, 3, ...\infty$and F denotes distribution functions corresponding to random variables $X_n$and X respectively. Suppose F_n converges weakly to F and further suppose $P(X = 0) = 1$. Examine whether $X_n \to X$ in probability. [8]

5. i) A transition matrix P is said to be doubly stochastic if $\sum_{i=1}^{k} p_{ij} = \sum_{j=1}^{k} p_{ij} = 1$ for every i and j. Suppose $lim_{n \to \infty} p_{ij}^n$ exists and it depends only on j and independent of i. Here p_{ij}^n denotes i, j^{th}element of P^n. Show that $\lim p_{ij}^n = 1/k$ for all j.
   ii) Consider a countable markov chain with $S = \{0, 1, 2, ...\}$ and probabilities $p(n, n+1) = p = 1 - p(n, 0)$.
   For which value of p, is this transient and recurrent? [3 + 7]

6. Examine whether the following statesments are true of false. Give reasons in either case:
i) Let F_n denote distribution function corresponding to Normal$(0, \frac{\sigma^2}{n})$. Then $F_n$converges weakly to a distribution function F.
ii) Let $R = (-\infty, \infty)$. Let $\mathscr{F} = \{A : A \subset R$ and either A or A' is countable$\}$. Then F is a field but not a σfield.
iii) Let (X, Y) follow a bivariate normal with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Then $E(X^2) + E(Y^2) - 2E(XY) \geq (E(X) - E(Y))^2$ [4 + 4 + 4]

For Brevity and Neatness............ [5]